

Anomalies in weak decays of H-like ions

Francesco Giacosa

*Institute of Physics, Jan Kochanowski University, ul. Swietokrzyska 15, 25-406 Kielce,
Poland*

Giuseppe Pagliara

*Dip. di Fisica e Scienze della Terra dell'Università di Ferrara and INFN Sez. di Ferrara,
Via Saragat 1, I-44100 Ferrara, Italy*

Abstract

We investigate the emergence of oscillations in the decay law of unstable systems. We discuss in particular the case of the so-called GSI anomaly seen in the electron capture decays of H-like ions and prove that such oscillations cannot be explained by neutrino oscillations. We then discuss how such anomalies could be intimately related to the decay law of unstable systems in the case in which their spectral function deviates from a Breit-Wigner shape.

Keywords: decay law, GSI anomaly, neutrinos

2010 MSC: 00-01, 99-00

1. Introduction

The decay of the H-like ions ^{140}Pr and ^{142}Pm via electron capture was measured at the GSI storage ring [1], where some peculiar oscillations in the decay law were found. In some works [2] these anomalies were linked to the phenomenon of neutrino oscillations. However, this interpretation has been also heavily disputed in other articles, see for instance [3].

In this work we first confirm, by using the Lee-Hamiltonian formalism [4] for the study of decays [5, 6], that neutrino oscillations cannot generate time modulations in the experimental set-up of the GSI experiment. Then, we discuss the emergence of a non-exponential decay law as a consequence of deviations from

the Breit-Wigner energy distribution. Indeed, a decay law which presents clear oscillations was measured in the decay thorough tunneling of sodium atoms in an accelerated optical potential [7], thus showing that the short-time deviations from the decay law are an experimental fact.

15 2. Neutrino oscillations: why they cannot generate time modulations

The process under study is schematically given by $M \rightarrow D + \nu_e$, where M stays for the H -like mother state (such as ^{140}Pr) and D for the daughter nucleus state (such as ^{140}Ce). Because of neutrino mixing ($\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2$) one obtains the decay into two channels: $M \rightarrow D + \nu_1$ and $M \rightarrow D + \nu_2$. The energy-momentum conservation implies that (in the first channel) $p = q_1 + k_1$, out of which (in the reference frame of the mother) $M_M^2 + m_{\nu_1}^2 - 2M_M E_{\nu_1} = M_D^2$. A similar expression holds in the second channel. Then, the energy difference is given by $\Delta E_\nu = E_{\nu_2} - E_{\nu_1} = (m_{\nu_2}^2 - m_{\nu_1}^2)/2M_M$. Now, if the decay amplitude *would* have an expression of the form

$$A \propto \cos\theta e^{-iE_{\nu_1}t} + \sin\theta e^{-iE_{\nu_2}t}, \quad (1)$$

then the square amplitude *would* read

$$|A|^2 \propto 1 + 2\cos\theta\sin\theta\cos(\Delta E_\nu t), \quad (2)$$

out of which oscillations with period $T = 2\pi/\Delta E_\nu$ emerge. Obviously, this is not a derivation of an oscillation formula. Actually, a straightforward calculation of the survival amplitude $a(t)$, as shown in the following, does **not** lead to an oscillatory amplitude of this kind. Anyway, it is suggestive that, if we use the
20 present value for the mass difference $\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{eV}^2$ [8], we obtain (by setting $M_M = 132 \text{ GeV}$) $T = 2\pi/\Delta E_\nu = 4\pi M_M/\Delta m_{21}^2 \simeq 14 \text{ s}$, which is *remarkably* close to the measured oscillation period $T_{\text{measured}} \simeq 8 \text{ s}$. This is the reason why neutrino oscillation has been considered appealing, even if it cannot hold.

We now turn to the Lee-Hamiltonian formalism to describe decays [4]. This formalism is equivalent to QFT at one-loop [6, 9, 10] (in most cases a good

approximation [11]), hence the correct formalism to describe decays. The basis of states that we consider is given by $\{|M\rangle, |D(\mathbf{k}), \nu_1(-\mathbf{k})\rangle, |D(\mathbf{k}), \nu_2(-\mathbf{k})\rangle\}$, where again $M \equiv$ mother (in the rest frame) and $D \equiv$ daughter. The Hamiltonian $H = H_0 + H_1$ reads:

$$\begin{aligned} H_0 &= M_0 |M\rangle \langle M| + \sum_{i=1,2} \int d\mathbf{k} \omega_i(\mathbf{k}) |D(\mathbf{k}), \nu_i(-\mathbf{k})\rangle \langle D(\mathbf{k}), \nu_i(-\mathbf{k})| , \\ H_1 &= \sum_{i=1,2} \int d\mathbf{k} \frac{g_i f_i(\mathbf{k})}{(2\pi)^{3/2}} (|M\rangle \langle D(\mathbf{k}), \nu_i(-\mathbf{k})| + \text{h.c.}) . \end{aligned} \quad (3)$$

The state $|D(\mathbf{k}), \nu_1(-\mathbf{k})\rangle$ ($|D(\mathbf{k}), \nu_2(-\mathbf{k})\rangle$) represents a two-particle state, a D with momentum \mathbf{k} and a neutrino with $-\mathbf{k}$. The energies read $\omega_i(\mathbf{k}) = \sqrt{\mathbf{k}^2 + M_D^2} + \sqrt{\mathbf{k}^2 + m_{\nu_i}^2}$, g_i are coupling constants and $f_i(k)$ are form factors. In the exponential limit, $f_i(k) = 1$, the time evolution reads:

$$e^{-iHt} |M\rangle = e^{-i(M_0 - i\Gamma/2)t} |M\rangle + \sum_{i=1,2} \int d\mathbf{k} b_i(\mathbf{k}, t) e^{-i\omega_i(\mathbf{k})t} |D(\mathbf{k}), \nu_i(-\mathbf{k})\rangle \quad (4)$$

where $b_i(\mathbf{k}, t) = \frac{g_i}{(2\pi)^{3/2}} \frac{e^{-i\omega_i(\mathbf{k})t} - e^{-i(M_0 - i\Gamma/2)t}}{\omega_i(\mathbf{k}) - M_0 + i\Gamma/2}$, see e.g. [12]. The survival probability amplitude of the state $|M\rangle$ is also in this case the usual exponential form $a(t) = \langle S| e^{-iHt} |S\rangle = e^{-i(M_0 - i\Gamma/2)t}$, hence the survival probability is $p(t) = e^{-\Gamma t}$ (with $\Gamma = \Gamma_1 + \Gamma_2$ and $\Gamma_i = g_i^2$). Alternatively, one can calculate the probability that the state has decayed between $(0, t)$, which is given by

$$w(t) = \int d\mathbf{k} \left[|b_1(\mathbf{k}, t)|^2 + |b_2(\mathbf{k}, t)|^2 \right] = 1 - e^{-\Gamma t} = 1 - p(t) , \quad (5)$$

where clearly no oscillations exist. If, instead, we evaluate the probability to measure the final state in a combination corresponding to a neutrino ν_e ,

$$|F_e(\mathbf{k})\rangle = \cos \theta |D(\mathbf{k}), \nu_1(-\mathbf{k})\rangle + \sin \theta |D(\mathbf{k}), \nu_2(-\mathbf{k})\rangle \quad (6)$$

²⁵ (for whatever value of \mathbf{k}), we find: $w_{\nu_e} = \int d\mathbf{k} \left[|\cos \theta b_1(\mathbf{k}, t) + \sin \theta b_2(\mathbf{k}, t)|^2 \right]$. In general, $w_{\nu_e}(t)$ will display some oscillations. Their intensity depend on the physical scales of the system. Similarly, we could evaluate the probability to find the final state in the orthogonal combination leading to $w_{\nu_\mu} = \int d\mathbf{k} \left[|-\sin \theta b_1(\mathbf{k}, t) + \cos \theta b_2(\mathbf{k}, t)|^2 \right]$. This expression also leads to oscillations.

30 But, if we do not distinguish among these two configurations (because we do
not measure neutrinos but only the mother and the daughter states) we have
to perform the sum and re-obtain the standard formula: $w_{\nu_e} + w_{\nu_\mu} = w(t) =$
 $1 - p(t) = 1 - e^{-\Gamma t}$. Again, the oscillations disappear. In the end, no mat-
ter how one designs the decay's measurement of the mother and the daughter
35 states: neutrino oscillations do not generate time modulations.

3. Non-exponential decay

In QM the exponential decay is only an (extremely good) approximation:
deviations are predicted at both short and late times after the creation of the
unstable system [5, 6, 13]. The very same effect has been confirmed in QFT [10].
40 Short-time deviations (related to a residual correlation between unstable system
and decay products) have been clearly demonstrated in cold atoms experiments
[7], while long-time deviations were measured in molecular decays [14].

In Ref. [15], we proposed that a similar deviation occurs in the electron-
capture decays of H-like ions due to the particular way in which the energy
eigenstate measurement is performed in a storage ring: no measurement on the
outgoing neutrino and long-lasting measurement of the daughter nucleus [1]. As
a simple model, we consider the case in which the form factors $f_i(k)$ generates
a Breit-Wigner $d_M(E) = N\Gamma [(E - M)^2 + \Gamma^2/4]^{-1}$ which is cut by a cut-off Λ
in an energy window centered at the value of the mass of the unstable system.
The corresponding survival probability

$$p(t) = \left| \int_{M_M - \Lambda}^{M_M + \Lambda} d_M(E) e^{-iEt} dE \right|^2 \quad (7)$$

shows an oscillating behavior with a period $T \sim 1/\Lambda$ (see plots in [15]).

In this interpretation, the physical origin of the cutoff is related (in a non
45 trivial way) to the time needed to measure the mass of the daughter nucleus,
which is of the order of 1sec. During this time interval the mother nucleus
and the decay products are still correlated and can therefore provide significant
deviations from the usual exponential decay law. As a consequence, changing

the detector would also affect the results (in particular, a detector with a higher
50 precision in the measurement of time would suppress the signal).

Within our model it is not possible to obtain oscillations in β^+ decays because the emitted positron is immediately adsorbed within the detector and the correlation between mother and daughter states is broken (similarly for electron capture decay experiments in which the nuclei are embedded in a metallic
55 matrix [16]).

4. Conclusions

The not-yet clarified GSI oscillations need experimental verification/falsification. Here, we have shown that neutrino oscillations cannot be responsible for them, but that deviations from the exponential decay law offer an interesting possibility. If the effect shall be confirmed, the microphysics of the measurement process
60 specifically used in the GSI experiment needs to be theoretically modelled in a better way.

References

- [1] Y. A. Litvinov *et al.*, Phys. Lett. B **664** (2008) 162; P. Kienle *et al.* [Two-
65 Body-Weak-Decays Collaboration], Phys. Lett. B **726** (2013) 638.
- [2] A. N. Ivanov and P. Kienle, Phys. Rev. Lett. **104** (2010) 159202; T. Faestermann *et al* Phys. Lett. B **672**, 227 (2009)
- [3] A. Merle, Phys. Rev. C **80** (2009) 054616; C. Giunti, Phys. Lett. B **665**
(2008) 92; A. G. Cohen, S. L. Glashow and Z. Ligeti, Phys. Lett. B **678**
70 (2009) 191.
- [4] T. D. Lee, Phys. Rev. **95** (1954) 1329-1334.
- [5] P. Facchi, H. Nakazato, S. Pascazio Phys. Rev. Lett. **86** (2001) 2699-2703.
- [6] F. Giacosa, Found. Phys. **42** (2012) 1262.

- [7] S. R. Wilkinson *et al*, Nature **387**, 575 (1997); M. C. Fischer, B. Gutiérrez-Medina and M. G. Raizen, Phys. Rev. Lett. **87**, 040402 (2001).
75
- [8] K. Abe *et al*. [Super-Kamiokande Collaboration], Phys. Rev. D **83** (2011) 052010.
- [9] F. Giacosa, G. Pagliara, Phys. Rev. **C76** (2007) 065204.
- [10] F. Giacosa, G. Pagliara, Mod. Phys. Lett. **A26** (2011) 2247-2259.
- [11] J. Schneitzler, T. Wolkanowski and F. Giacosa, Nucl. Phys. B **888** (2014) 287.
80
- [12] F. Giacosa, Phys. Rev. A **88**, **052131** (2013).
- [13] L. Fonda, G. C. Ghirardi and A. Rimini, Rept. Prog. Phys. **41** (1978) 587.
- [14] C. Rothe, S. I. Hintschich, A. P. Monkman, Phys. Rev. Lett. **96**
85 (2006)163601.
- [15] F. Giacosa, G. Pagliara , Quant. Matt. **2** (2013) 54; F. Giacosa, G. Pagliara, PoS BORMIO **2012** (2012) 028.
- [16] P. A. Vetter et al, Phys. Lett. B **670** (2008) 196.